

An extension of tribimaximal lepton mixing

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Abstract

Harrison, Perkins and Scott have proposed simple charged lepton and neutrino mass matrices that lead to the tribimaximal mixing U_{TBM} . We consider in this work an extension of the mass matrices so that the leptonic mixing matrix becomes $U_{\text{PMNS}} = V_L^{\ell\dagger} U_{\text{TBM}} W$, where V_L^{ℓ} is a unitary matrix needed to diagonalize the charged lepton mass matrix and W measures the deviation of the neutrino mixing matrix from the bimaximal form. Hence, corrections to U_{TBM} arise from both charged lepton and neutrino sectors. Following our previous work to assume a Qin-Ma-like parametrization V_{QM} for the charged lepton mixing matrix V_L^{ℓ} in which the CP -odd phase is approximately maximal, we study the phenomenological implications in two different scenarios: $V_L^{\ell} = V_{\text{QM}}^{\dagger}$ and $V_L^{\ell} = V_{\text{QM}}$. We find that the latter is more preferable, though both scenarios are consistent with the data within 3σ ranges. The predicted reactor neutrino mixing angle θ_{13} in both scenarios is consistent with the recent T2K and MINOS data. The leptonic CP violation characterized by the Jarlskog invariant J_{CP} is generally of order 10^{-2} .

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I. INTRODUCTION

The large values of the solar (θ_{12}) and atmospheric (θ_{23}) mixing angles may be telling us about some new symmetries of leptons not presented in the quark sector and may provide a clue to the nature of the quark-lepton physics beyond the standard model. If there exists such a flavor symmetry in Nature, the tribimaximal (TBM) [1] pattern for the neutrino mixing will be a good zeroth order approximation to reality :

$$\sin^2 \theta_{12} = \frac{1}{3} , \quad \sin^2 \theta_{23} = \frac{1}{2} , \quad \sin \theta_{13} = 0 . \quad (1)$$

For example, in a well-motivated extension of the standard model through the inclusion of A_4 discrete symmetry, the TBM pattern comes out in a natural way in the work of [2]. Although such a flavor symmetry is realized in Nature leading to exact TBM, in general there may be some deviations from TBM. Recent data of the T2K [3] and MINOS [4] Collaborations and the analysis based on global fits [5, 6] of neutrino oscillations enter into a new phase of precise measurements of the neutrino mixing angles and mass-squared differences, indicating that the TBM mixing for three flavors of leptons should be modified. In the weak eigenstate basis, the Yukawa interactions in both neutrino and charged lepton sectors and the charged gauge interaction can be written as

$$-\mathcal{L} = \frac{1}{2} \overline{\nu_L} \mathcal{M}_\nu (\nu_L)^c + \overline{\ell_L} m_\ell \ell_R + \frac{g}{\sqrt{2}} W_\mu^- \overline{\ell_L} \gamma^\mu \nu_L + \text{H.c.} . \quad (2)$$

When diagonalizing the neutrino and charged lepton mass matrices $U_\nu^\dagger \mathcal{M}_\nu U_\nu^* = \text{diag}(m_1, m_2, m_3)$, $U_L^\dagger m_\ell U_R = \text{diag}(m_e, m_\mu, m_\tau)$, one can rotate the neutrino and charged lepton fields from the weak eigenstates to the mass eigenstates $\nu_L \rightarrow U_\nu^\dagger \nu_L$, $\ell_{L(R)} \rightarrow U_{L(R)}^\dagger \ell_{L(R)}$. Then we obtain the leptonic 3×3 unitary mixing matrix $U_{\text{PMNS}} = U_L^\dagger U_\nu$ from the charged current term in Eq. (2). In the standard parametrization of the leptonic mixing matrix U_{PMNS} , it is expressed in terms of three mixing angles and three CP -odd phases (one for the Dirac neutrino and two for the Majorana neutrino) [7]

$$U_{\text{PMNS}} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{CP}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} P_\nu , \quad (3)$$

where $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$, and $P_\nu = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, 1)$ is a diagonal phase matrix which contains two CP -violating Majorana phases, one (or a combination) of which can be

in principle explored through the neutrinoless double beta ($0\nu 2\beta$) decay [8]. For the global fits of the available data from neutrino oscillation experiments, we quote two recent analyses: one by Gonzalez-Garcia *et al.* [5]

$$\begin{aligned}\sin^2 \theta_{12} &= 0.319^{+0.016 (+0.053)}_{-0.016 (-0.046)} , & \sin \theta_{13} &= 0.097^{+0.052}_{-0.050} (\leq 0.217) , \\ \sin^2 \theta_{23} &= 0.462^{+0.082 (+0.185)}_{-0.050 (-0.124)} ,\end{aligned}\tag{4}$$

in 1σ (3σ) ranges, or equivalently

$$\theta_{12} = 34.4^{+1.0^\circ (+3.2^\circ)}_{-1.0^\circ (-2.9^\circ)} , \quad \theta_{23} = 42.8^{+4.7^\circ (+10.7^\circ)}_{-2.9^\circ (-7.3^\circ)} , \quad \theta_{13} = 5.6^{+3.0^\circ (+6.9^\circ)}_{-2.9^\circ (-5.6^\circ)} ,\tag{5}$$

and the other given by Fogli *et al.* with new reactor neutrino fluxes [6]:

$$\begin{aligned}\sin^2 \theta_{12} &= 0.312^{+0.017 (+0.052)}_{-0.006 (-0.047)} , & \sin^2 \theta_{13} &= 0.025^{+0.007 (+0.025)}_{-0.007 (-0.020)} , \\ \sin^2 \theta_{23} &= 0.42^{+0.08 (+0.22)}_{-0.03 (-0.08)} ,\end{aligned}\tag{6}$$

corresponding to

$$\theta_{12} = 34.0^{+1.0^\circ (+3.2^\circ)}_{-1.0^\circ (-3.0^\circ)} , \quad \theta_{23} = 40.4^{+4.6^\circ (+12.7^\circ)}_{-1.3^\circ (-4.7^\circ)} , \quad \theta_{13} = 9.1^{+1.2^\circ (+3.8^\circ)}_{-1.4^\circ (-5.0^\circ)} .\tag{7}$$

The analysis by Fogli *et al.* includes the T2K [3] and MINOS [4] results. The T2K Collaboration [3] has announced that the value of θ_{13} is non-zero at 90% C.L. with the ranges

$$0.03 (0.04) \leq \sin^2 2\theta_{13} \leq 0.28 (0.34) ,\tag{8}$$

or

$$4.99^\circ (5.77^\circ) \leq \theta_{13} \leq 15.97^\circ (17.83^\circ)\tag{9}$$

for $\delta_{CP} = 0$, $\sin^2 2\theta_{23} = 1$ and the normal (inverted) neutrino mass hierarchy. The MINOS Collaboration found

$$\sin^2 2\theta_{13} \leq 0.12 (0.20) ,\tag{10}$$

with a best fit of

$$\sin^2 2\theta_{13} = 0.041^{+0.047}_{-0.031} (0.079^{+0.071}_{-0.053}) ,\tag{11}$$

for $\delta_{CP} = 0$, $\sin^2 2\theta_{23} = 1$ and the normal (inverted) neutrino mass hierarchy. The experimental result of non-zero $|U_{e3}| \equiv \sin \theta_{13}$ implies that the TBM pattern should be modified. However, properties related to the leptonic CP violation remain completely unknown yet.

The trimaximal neutrino mixing was first proposed by Cabibbo [9]¹ (see also [10])

$$V_C = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega \\ 1 & 1 & 1 \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad (12)$$

with $\omega = e^{i2\pi/3}$ being a complex cube-root of unity. This mixing matrix has maximal CP violation with the Jarlskog invariant $|J_{CP}| = 1/(6\sqrt{3})$. However, this trimaximal mixing pattern has been ruled out by current experimental data on neutrino oscillations. In their original work, Harrison, Perkins and Scott (HPS) [1] proposed to consider the simple mass matrices

$$M_\ell^2 = \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix}, \quad M_\nu^2 = \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix}, \quad (13)$$

that can lead to the tribimaximal mixing, where a, x, y and z are real parameters,² $M_\ell^2 \equiv m_\ell m_\ell^\dagger$ and $M_\nu^2 \equiv \mathcal{M}_\nu \mathcal{M}_\nu^\dagger$. The mass matrices are diagonalized by the trimaximal matrix V_C for charged lepton fields and the bimaximal matrix U_{BM} defined below for neutrino fields, that is, $V_C^\dagger M_\ell^2 V_C = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$ and $U_{\text{BM}}^\dagger M_\nu^2 U_{\text{BM}} = \text{diag}(m_1^2, m_2^2, m_3^2)$. The combination of trimaximal and bimaximal matrices leads to the so-called TBM mixing matrix:

$$U_{\text{TBM}} = V_C^\dagger U_{\text{BM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{i}{\sqrt{2}} \end{pmatrix} \quad \text{with } U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (14)$$

¹ The matrix originally given by Cabibbo was in the form

$$V_C = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^* \\ 1 & \omega^* & \omega \end{pmatrix}.$$

If one considers A_4 discrete symmetry, it will have two subgroups, namely, Z_2 and Z_3 . The trimaximal matrix given in Eq. (12) is obtained under Z_3 .

² Different from the choice of HPS, the matrix element y in Eq. (13) can be in general introduced as complex: e.g., $(M_\nu^2)_{13} = y$ and $(M_\nu^2)_{31} = y^*$. This case has been considered by Xing [11] who pointed out that the off-diagonal terms in U_{BM} will acquire a phase from the complex y . It has the interesting implication that a nonzero $\sin\theta_{13}$ will result from the phase of y . However, the corresponding Jarlskog invariant is exactly zero and the absence of intrinsic CP violation makes this possibility less interesting.

It is clear by now that the tribimaximal mixing is not consistent with the recent experimental data on the reactor mixing angle θ_{13} because of the vanishing matrix element U_{e3} in U_{TBM} .

In this work we consider an extension of the tribimaximal mixing by considering small perturbations to the mass matrices M_ℓ^2 and M_ν^2 which we will call $M_\ell'^2$ and $M_\nu'^2$, respectively (see Eq. (15) below) so that $U_\nu = U_{\text{BM}}W$ is no longer in the bimaximal form and $U_L = V_C V_L^\ell$ deviates from the trimaximal structure, where V_L^ℓ is the unitary matrix needed to diagonalize the matrix $V_C^\dagger M_\ell'^2 V_C$. As a consequence, $U_{\text{PMNS}} = U_L^\dagger U_\nu = V_L^{\ell\dagger} U_{\text{TBM}} W = U_{\text{TBM}} +$ small perturbations. Hence, the corrections to the TBM pattern arise from both charged lepton and neutrino sectors. Inspired by the T2K and MINOS measurements of a sizable reactor angle θ_{13} , there exist in the literature intensive studies of possible deviations from the exact TBM pattern. However, most of these investigations were focused on the modification of TBM arising from either the neutrino sector [12] or the charged lepton part [13, 14], but not both simultaneously.

The paper is organized as follows. In Sec. II, we set up the model by making a general extension to the charged lepton and neutrino mass matrices. Then in Sec. III we study the phenomenological implications by considering two different scenarios for the charged lepton mixing matrix. Our conclusions are summarized in Sec. IV.

II. A SIMPLE AND REALISTIC EXTENSION

In order to discuss the deviation from the TBM mixing, let us consider a simple and general extension of the original proposal by HPS given in Eq. (13), by taking into account perturbative effects on the mass matrices M_ℓ^2 and M_ν^2 . The generalized mass matrices $M_f'^2$ and $M_\nu'^2$ can be introduced as ³

$$M_f'^2 = \begin{pmatrix} a + g_3 & b + \chi_3 & b^* + \chi_2^* \\ b^* + \chi_3^* & a + g_2 & b + \chi_1 \\ b + \chi_2 & b^* + \chi_1^* & a + g_1 \end{pmatrix}, \quad M_\nu'^2 = m_0^2 \begin{pmatrix} x' & 0 & y' \\ 0 & 1 & 0 \\ y' & 0 & x' + \rho \end{pmatrix}, \quad (15)$$

³ TBM could be obtained in models with different discrete symmetries, such as S_3, A_4, S_4, A_5 , dihedral groups, \dots , etc. By considering higher order and radiative effects, the matrices in Eq. (15) can be realized. For example, we have shown in Ref. [15] that these matrices can be obtained by introducing dimension-5 operators to the Lagrangians.

where $M_f'^2$ and $M_\nu'^2$ are defined as the hermitian square of the mass matrices $M_f'^2 \equiv m_f' m_f'^\dagger$ and $M_\nu'^2 \equiv \mathcal{M}_\nu' \mathcal{M}_\nu'^\dagger$, respectively, with the subscript f denoting charged fermion fields (charged leptons or quarks). Due to the hermiticity of $M_f'^2$ and $M_\nu'^2$, the parameters $a, g_{1,2,3}, m_0^2, x', y', \rho$ are real, while b and $\chi_{1,2,3}$ are complex. The parameters $g_{1,2,3}, \chi_{1,2,3}$ and ρ represent small perturbations. Note that the (11), (13), (22) elements (*i.e.*, $m_0^2 x', m_0^2 y'$ and m_0^2) in $M_\nu'^2$ are assumed to contain any perturbative effects on the elements x, y , and z in M_ν^2 , respectively. For simplicity, it is assumed that y' is real just as the other elements in $M_\nu'^2$ and the vanishing off-diagonal elements in M_ν^2 remain zeros in $M_\nu'^2$.

The parameters a and b are encoded in [1] as

$$a = \frac{\tilde{m}_{f_1}^2}{3} + \frac{\tilde{m}_{f_2}^2}{3} + \frac{\tilde{m}_{f_3}^2}{3}, \quad b = \frac{\tilde{m}_{f_1}^2}{3} + \frac{\tilde{m}_{f_2}^2 \omega^2}{3} + \frac{\tilde{m}_{f_3}^2 \omega}{3}, \quad (16)$$

where the subscript f_i indicates a generation of charged fermion field, and \tilde{m}_{f_i} represents a bare mass of f_i , for example, $\tilde{m}_{f_1} = \tilde{m}_e \ll \tilde{m}_{f_2} = \tilde{m}_\mu \ll \tilde{m}_{f_3} = \tilde{m}_\tau$ for charged lepton fields.

We first discuss the hermitian square of the neutrino mass matrix, $M_\nu'^2$, in Eq. (15). It can be diagonalized by

$$U_\nu = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} P_\nu = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} W, \quad (17)$$

with

$$\tan 2\theta = -\frac{2y'}{\rho} \quad (18)$$

and

$$W = \begin{pmatrix} (\cos \theta + \sin \theta)/\sqrt{2} & 0 & (\cos \theta - \sin \theta)/\sqrt{2} \\ 0 & 1 & 0 \\ -(\cos \theta - \sin \theta)/\sqrt{2} & 0 & (\cos \theta + \sin \theta)/\sqrt{2} \end{pmatrix} P_\nu, \quad (19)$$

where the diagonal phase matrix P_ν contains two additional phases, which can be absorbed into the neutrino mass eigenstate fields. For a small perturbation $|\rho|$ ($\ll |x'|$), the mixing parameter θ can be expressed in terms of

$$\theta = \pi/4 + \epsilon \quad \text{with} \quad |\epsilon| \ll 1. \quad (20)$$

W is then reduced to

$$W = \begin{pmatrix} \cos \epsilon & 0 & -\sin \epsilon \\ 0 & 1 & 0 \\ \sin \epsilon & 0 & \cos \epsilon \end{pmatrix} P_\nu . \quad (21)$$

The neutrino mass eigenvalues are obtained as

$$m_1^2 = m_0^2(x' + \rho \sin^2 \theta + y' \sin 2\theta), \quad m_2^2 = m_0^2, \quad m_3^2 = m_0^2(x' + \rho \cos^2 \theta - y' \sin 2\theta) \quad (22)$$

and their differences are given by

$$\begin{aligned} \Delta m_{21}^2 &\equiv m_2^2 - m_1^2 = m_0^2 \left(1 - x' + \rho \frac{1 - \sin 2\epsilon}{2 \sin 2\epsilon} \right), \\ \Delta m_{31}^2 &\equiv m_3^2 - m_1^2 = m_0^2 \frac{2\rho}{\sin 2\epsilon}, \end{aligned} \quad (23)$$

from which we have a relation $\Delta m_{21}^2 - \frac{1}{4}\Delta m_{31}^2 \simeq m_2^2(1 - x')$. It is well known that the sign of Δm_{21}^2 is positive due to the requirement of the Mikheyev-Smirnov-Wolfenstein resonance for solar neutrinos. The sign of Δm_{31}^2 depends on that of $\rho/\sin 2\epsilon$: $\Delta m_{31}^2 > 0$ for the normal mass spectrum and $\Delta m_{31}^2 < 0$ for the inverted one. The quantities $m_1^2, m_2^2, m_3^2, \theta$ (or ϵ) are determined by the four parameters m_0^2, x', y', ρ , while the Majorana phases in Eq. (17) are hidden in the squared mass eigenvalues.

We next turn to the hermitian square of the mass matrix for charged fermions in Eq. (15). This modified charged fermion mass matrix is no longer diagonalized by V_C

$$V_C^\dagger M_f'^2 V_C = \begin{pmatrix} m_a^2 + \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{12}^* & m_b^2 + \eta_{22} & \eta_{23} \\ \eta_{13}^* & \eta_{23}^* & m_c^2 + \eta_{33} \end{pmatrix}, \quad (24)$$

where

$$m_a^2 = a + b + b^*, \quad m_b^2 = a + b \omega + b^* \omega^2, \quad m_c^2 = a + b \omega^2 + b^* \omega, \quad (25)$$

corresponding to $\tilde{m}_{f_1}^2, \tilde{m}_{f_2}^2, \tilde{m}_{f_3}^2$, respectively, and η_{ij} is composed of the combinations of $g_{1,2,3}$ and $\chi_{1,2,3}$. To diagonalize $V_C^\dagger M_f'^2 V_C = V_L^f \text{diag}(m_{f_1}^2, m_{f_2}^2, m_{f_3}^2) V_L^{f\dagger}$, we need an additional matrix V_L^f which can be, in general, parametrized in terms of three mixing angles and six phases:

$$V_L^f = \begin{pmatrix} c_2 c_3 & c_2 s_3 e^{i\alpha_3} & s_2 e^{i\alpha_2} \\ -c_1 s_3 e^{-i\alpha_3} - s_1 s_2 c_3 e^{i(\alpha_1 - \alpha_2)} & c_1 c_3 - s_1 s_2 s_3 e^{i(\alpha_1 - \alpha_2 + \alpha_3)} & s_1 c_2 e^{i\alpha_1} \\ s_1 s_3 e^{-i(\alpha_1 + \alpha_3)} - c_1 s_2 c_3 e^{-i\alpha_2} & -s_1 c_3 e^{-i\alpha_1} - c_1 s_2 s_3 e^{i(\alpha_3 - \alpha_2)} & c_1 c_2 \end{pmatrix} P_f, \quad (26)$$

where $s_i \equiv \sin \theta_i$, $c_i \equiv \cos \theta_i$ and a diagonal phase matrix $P_f = \text{diag}(e^{i\xi_1}, e^{i\xi_2}, e^{i\xi_3})$ which can be rotated away by the phase redefinition of left-charged fermion fields. The charged fermion mixing matrix now reads $U_L = V_C V_L^f$.

Finally, we arrive at the general expression for the leptonic mixing matrix

$$U_{\text{PMNS}} = U_L^\dagger U_\nu = V_L^{\ell\dagger} U_{\text{TBM}} W . \quad (27)$$

A simple and general extension of the mass matrices given in Eq. (15) thus leads to two possible sources of corrections to the tribimaximal mixing: V_L^ℓ measures the deviation of the charged lepton mixing matrix from the trimaximal form and W characterizes the departure of the neutrino mixing from the bimaximal one. The charged lepton mass matrix in Eq. (15) or (24) has 12 free parameters. Three of them are replaced by the phases $\xi_{1,2,3}$ in Eq. (26) which can be eliminated by a redefinition of the physical charged lepton fields. The remaining 9 parameters can be expressed in terms of $m_e, m_\mu, m_\tau, \theta_1, \theta_2, \theta_3, \alpha_1, \alpha_2, \alpha_3$. From Eqs. (24) and (26) the mixing angles and phases can be expressed as

$$\begin{aligned} \theta_1 &\simeq \frac{|\eta_{23}|}{\tilde{m}_\tau^2}, & \theta_2 &\simeq \frac{|\eta_{13}|}{\tilde{m}_\tau^2}, & \theta_3 &\simeq \frac{|\eta_{12}|}{\tilde{m}_\mu^2}, & \alpha_1 &= \arg(\eta_{23}), \\ \alpha_2 &\simeq \frac{1}{2} \arg(\eta_{23}) + \arg(\eta_{13}), & \alpha_3 &\simeq \frac{1}{2} [\arg(\eta_{13}) - \arg(\eta_{23})] + \arg(\eta_{12}), \end{aligned} \quad (28)$$

with the condition $\tilde{m}_{f_2}^2 \gg \eta_{22}, \eta_{11}$. In the charged fermion sector, there is a qualitative feature that distinguishes the neutrino sector from the charged fermion one. The mass spectrum of the charged leptons exhibits a similar hierarchical pattern to that of the down-type quarks, unlike that of the up-type quarks which show a much stronger hierarchical pattern. For example, in terms of the Cabibbo angle $\lambda \equiv \sin \theta_C \approx |V_{us}|$, the fermion masses scale as $(m_e, m_\mu) \approx (\lambda^5, \lambda^2) m_\tau$, $(m_d, m_s) \approx (\lambda^4, \lambda^2) m_b$ and $(m_u, m_c) \approx (\lambda^8, \lambda^4) m_t$. This may lead to two implications: (i) the Cabibbo-Kobayashi-Maskawa (CKM) matrix [16] is mainly governed by the down-type quark mixing matrix, and (ii) the charged lepton mixing matrix is similar to that of the down-type quark one. Therefore, we shall assume that (i) $V_{\text{CKM}} = V_L^{d\dagger}$ and $V_L^u = \mathbf{1}$, where V_L^d (V_L^u) is associated with the diagonalization of the down-type (up-type) quark mass matrix and $\mathbf{1}$ is a 3×3 unit matrix, and (ii) the charged lepton mixing matrix V_L^ℓ has the same structure as the CKM matrix, that is, $V_L^{\ell\dagger} = V_{\text{CKM}}$ or V_{CKM}^\dagger .

Recently, we have proposed a simple *ansatz* for the charged lepton mixing matrix V_L^ℓ , namely, it has the Qin-Ma-like parametrization in which the CP -odd phase is approximately maximal [13]. Armed with this *ansatz*, we notice that the 6 parameters $\theta_1, \theta_2, \theta_3, \alpha_1, \alpha_2, \alpha_3$

in V_L^ℓ are reduced to four independent ones f, h, λ, δ . It has the advantage that the TBM predictions of $\sin^2 \theta_{23} = 1/2$ and especially $\sin^2 \theta_{12} = 1/3$ will not be spoiled and that a sizable reactor mixing angle θ_{13} and a large Dirac CP -odd phase are obtained in the mixing $U_{\text{PMNS}} = V_L^{\ell\dagger} U_{\text{TBM}}$. The Qin-Ma (QM) parametrization of the quark CKM matrix is a Wolfenstein-like parametrization and can be expanded in terms of the small parameter λ [17]. However, unlike the original Wolfenstein parametrization [18], the QM one has the advantage that its CP -odd phase δ is manifested in the parametrization and is near maximal, *i.e.*, $\delta \sim 90^\circ$. This is crucial for a viable neutrino phenomenology. It should be stressed that one can also use any parametrization for the CKM matrix as a starting point. As shown in [19], one can adjust the phase differences in the diagonal phase matrix P_f in Eq. (26) in such a way that the prediction of $\sin^2 \theta_{12}$ will not be considerably affected.

For $V_L^{\ell\dagger} = V_{\text{QM}}$, the QM parametrization [13, 17] can be obtained from Eq. (26) by the replacements $s_1 e^{i\alpha_1} = -(f + h e^{-i\delta})\lambda^2$, $s_2 = f\lambda^3$, $s_3 = \lambda$, $\alpha_2 = \delta$, $\alpha_3 = \delta - \pi$:

$$V_L^{f\dagger} = P_f^* \begin{pmatrix} 1 - \lambda^2/2 & \lambda e^{i\delta} & h\lambda^3 \\ -\lambda e^{-i\delta} & 1 - \lambda^2/2 & (f + h e^{-i\delta})\lambda^2 \\ f\lambda^3 e^{-i\delta} & -(f + h e^{i\delta})\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (29)$$

On the other hand, for $V_L^\ell = V_{\text{QM}}$ the QM parametrization is obtained by the replacements $s_1 e^{i\alpha_1} = (f + h e^{-i\delta})\lambda^2$, $s_2 = h\lambda^3$, $s_3 = \lambda$, $\alpha_2 = 0$, $\alpha_3 = \delta$:

$$V_L^f = \begin{pmatrix} 1 - \lambda^2/2 & \lambda e^{i\delta} & h\lambda^3 \\ -\lambda e^{-i\delta} & 1 - \lambda^2/2 & (f + h e^{-i\delta})\lambda^2 \\ f\lambda^3 e^{-i\delta} & -(f + h e^{i\delta})\lambda^2 & 1 \end{pmatrix} P_f + \mathcal{O}(\lambda^4), \quad (30)$$

where the superscript f denotes d (down-type quarks) or ℓ (charged leptons). From the global fits to the quark mixing matrix given by [20] we obtain

$$f = 0.749_{-0.037}^{+0.034}, \quad h = 0.309_{-0.012}^{+0.017}, \quad \lambda = 0.22545 \pm 0.00065, \quad \delta = (89.6_{-0.86}^{+2.94})^\circ. \quad (31)$$

Because of the freedom of the phase redefinition for the quark fields, we have shown in [21] that the QM parametrization is indeed equivalent to the Wolfenstein one in the quark sector.

Finally, the leptonic mixing parameters $(\theta_{23}, \theta_{12}, \theta_{13}, \delta_{CP})$ except Majorana phases can be expressed in terms of five parameters θ (or ϵ), δ, f, h, λ , the last four being the QM parameters in the lepton sector. If we further assume that all the QM parameters except δ

have the same values in both the CKM and PMNS matrices, then only two free parameters left in the lepton mixing matrix are ϵ and δ . If δ is fixed to be the same as the CKM one, then there will be only one free parameter ϵ in our calculation. In the next section, we shall study the dependence of the mixing angles $\sin^2 \theta_{23}$, $\sin^2 \theta_{12}$, $\sin \theta_{13}$ and the Jarlskog invariant J_{CP} on δ and ϵ .

To make our point clearer, let us summarize the reduction of the number of independent parameters in this work. In the leptonic sector, we start with 16 free parameters (12 from the charged lepton mass matrix $M_\ell'^2$ and 4 from the neutrino mass matrix $M_\nu'^2$) as shown in Eq. (15). Among the 12 parameters from $M_\ell'^2$, three phases can be rotated away by the redefinition of the charged lepton fields. The remaining 9 parameters correspond to three charged lepton masses ($m_{e,\mu,\tau}$) and six angles in the charged lepton mixing matrix V_L^ℓ as shown in Eq. (26), while the 4 parameters from $M_\nu'^2$ correspond to three neutrino masses ($m_{1,2,3}$) plus one angle (θ or ϵ) in the neutrino mixing matrix U_ν as shown in Eq. (17) or (21). With our *ansatz* for V_L^ℓ discussed before, the 6 angles in V_L^ℓ are reduced to four QM parameters (f, h, λ, δ). Thus, the number of parameters finally becomes five (f, h, λ, δ plus θ (or ϵ)), except for the six lepton masses. Under the further assumption of the QM parameters f, h, λ having the same values in both the CKM and PMNS matrices, these five parameters are reduced to only two ones δ and ϵ .

III. NEUTRINO PHENOMENOLOGY

We now proceed to discuss the low energy neutrino phenomenology with the neutrino mixing matrix U_ν (see Eq. (17)) characterized by the mixing angle θ or the small parameter ϵ and the charged lepton mixing matrix $U_L = V_C V_L^\ell$ in which V_L^ℓ is assumed to have the similar expression as the QM parametrization [13, 17] given by V_{QM}^\dagger or V_{QM} (see Eq. (29) and Eq. (30), respectively). The lepton mixing matrix thus has the form

$$U_{\text{PMNS}} = \begin{cases} V_{\text{QM}} U_{\text{TBM}} W & \text{for } V_L^{\ell\dagger} = V_{\text{QM}}, \\ V_{\text{QM}}^\dagger U_{\text{TBM}} W & \text{for } V_L^\ell = V_{\text{QM}}. \end{cases} \quad (32)$$

Therefore, the corrections to the TBM matrix within our framework arise from the charged lepton mixing matrix V_L^ℓ characterized by the parameters f, h, λ, δ and the matrix W specified by the parameter ϵ whose size is strongly constrained by the recent T2K data. Indeed, the parameters λ , f , h and δ in the lepton sector are *a priori* not necessarily the same

as that in the quark sector. Hereafter, we shall use the central values in Eq. (31) of the parameters (λ, f, h) for our numerical calculations.

In the following we consider both cases:

(i) $V_L^{\ell\dagger} = V_{\text{QM}}$

With the help of Eqs. (14) and (29), the leptonic mixing matrix corrected by the replacements $V_C \rightarrow U_L = V_C V_L^\ell = V_C V_{\text{QM}}^\dagger$ and $U_\nu(\pi/4) \rightarrow U_\nu(\pi/4 + \epsilon)$, can be written, up to order of λ^3 and ϵ^2 , as

$$\begin{aligned}
U_{\text{PMNS}}^{(i)} &= U_L^\dagger U_\nu(\pi/4 + \epsilon) = V_{\text{QM}} U_{\text{TBM}} W \\
&= U_{\text{TBM}} + \epsilon \begin{pmatrix} -\frac{\epsilon}{2}\sqrt{\frac{2}{3}} & 0 & \sqrt{\frac{2}{3}} \\ \frac{i}{\sqrt{2}} + \frac{\epsilon}{2\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} + \frac{i\epsilon}{2\sqrt{2}} \\ -\frac{i}{\sqrt{2}} + \frac{\epsilon}{2\sqrt{6}} & 0 & -\frac{1}{\sqrt{6}} - \frac{i\epsilon^2}{2\sqrt{2}} \end{pmatrix} \\
&+ \lambda \begin{pmatrix} -\frac{e^{i\delta} + \lambda + h\lambda^2}{\sqrt{6}} & \frac{e^{i\delta} - \frac{\lambda}{2} - h\lambda^2}{\sqrt{3}} & -\frac{i(e^{i\delta} - h\lambda^2)}{\sqrt{2}} \\ \frac{-2e^{-i\delta} + (\frac{1}{2} - f - he^{-i\delta})\lambda}{\sqrt{6}} & \frac{e^{-i\delta} + (\frac{1}{2} - f - he^{-i\delta})\lambda}{\sqrt{3}} & \frac{i(\frac{1}{2} + f + he^{-i\delta})\lambda}{\sqrt{2}} \\ \frac{(f + he^{i\delta})\lambda + 2fe^{-i\delta}\lambda^2}{\sqrt{6}} & -\frac{(f + he^{i\delta})\lambda + fe^{-i\delta}\lambda^2}{\sqrt{3}} & \frac{i(f + he^{i\delta})\lambda}{\sqrt{2}} \end{pmatrix} \\
&- \lambda\epsilon \begin{pmatrix} -\frac{i(e^{i\delta} - h\lambda^2)}{\sqrt{2}} - \epsilon\frac{e^{i\delta} + \lambda + h\lambda^2}{2\sqrt{6}} & 0 & \frac{e^{i\delta} + \lambda + h\lambda^2}{\sqrt{6}} - \epsilon\frac{i(e^{i\delta} - h\lambda^2)}{2\sqrt{2}} \\ \frac{i(\frac{1}{2} + f + he^{-i\delta})\lambda}{\sqrt{2}} - \epsilon\frac{2e^{-i\delta} + (f + he^{-i\delta} - \frac{1}{2})\lambda}{2\sqrt{6}} & 0 & \frac{2e^{-i\delta} + (f + he^{-i\delta} - \frac{1}{2})\lambda}{\sqrt{6}} + \epsilon\frac{i(f + he^{-i\delta} + \frac{1}{2})\lambda}{2\sqrt{2}} \\ \frac{i(f + he^{i\delta})\lambda}{\sqrt{2}} + \epsilon\frac{(f + he^{i\delta})\lambda + 2fe^{-i\delta}\lambda^2}{2\sqrt{6}} & 0 & -\frac{(f + he^{i\delta})\lambda + 2\lambda^2 fe^{i\delta}}{\sqrt{6}} + \epsilon\frac{i(f + he^{i\delta})\lambda}{2\sqrt{2}} \end{pmatrix}.
\end{aligned} \tag{33}$$

Note that $U_{\text{PMNS}}^{(i)}$ here contains five independent parameters $(\lambda, h, f, \delta \text{ and } \epsilon)$.⁴ By rephasing the lepton and neutrino fields $e \rightarrow e e^{i\alpha_1}$, $\mu \rightarrow \mu e^{i\beta_1}$, $\tau \rightarrow \tau e^{i\beta_2}$ and $\nu_2 \rightarrow \nu_2 e^{i(\alpha_1 - \alpha_2)}$, the PMNS matrix is recast to

$$U_{\text{PMNS}} = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}|e^{-i(\alpha_1 - \alpha_3)} \\ U_{\mu 1}e^{-i\beta_1} & U_{\mu 2}e^{i(\alpha_1 - \alpha_2 - \beta_1)} & |U_{\mu 3}| \\ U_{\tau 1}e^{-i\beta_2} & U_{\tau 2}e^{i(\alpha_1 - \alpha_2 - \beta_2)} & |U_{\tau 3}| \end{pmatrix} P_\nu, \tag{34}$$

where $U_{\alpha j}$ is an element of the PMNS matrix with $\alpha = e, \mu, \tau$ corresponding to the lepton flavors and $j = 1, 2, 3$ to the light neutrino mass eigenstates. In Eq. (34) the phases defined as $\alpha_1 = \arg(U_{e1})$, $\alpha_2 = \arg(U_{e2})$, $\alpha_3 = \arg(U_{e3})$, $\beta_1 = \arg(U_{\mu 3})$ and $\beta_2 = \arg(U_{\tau 3})$ have the

⁴ Our previous work [13] corresponds to case (i) with $\epsilon = 0$.

expressions:

$$\begin{aligned}
\alpha_1 &= \tan^{-1} \left(\frac{\lambda \{ \sqrt{3}(\epsilon^2 - 2) \sin \delta + 6\epsilon \cos \delta - 6h\epsilon\lambda^2 \}}{\sqrt{3}(2 - \epsilon^2)(2 - \lambda^2 - h\lambda^3) + \sqrt{3}(\epsilon^2 - 2)\lambda \cos \delta - 6\epsilon\lambda \sin \delta} \right) , \\
\alpha_2 &= \tan^{-1} \left(\frac{\lambda \sin \delta}{1 + \lambda \cos \delta - \frac{\lambda^2}{2} + h\lambda^3} \right) , \\
\alpha_3 &= \tan^{-1} \left(\frac{\lambda \{ 2\sqrt{3}\epsilon \sin \delta + 3(2 - \epsilon^2) \cos \delta - 3h(2 - \epsilon^2)\lambda^3 \}}{3\lambda(\epsilon^2 - 2) \sin \delta - 2\sqrt{3}\epsilon(2 - \lambda^2 - \lambda \cos \delta - h\lambda^3)} \right) , \\
\beta_1 &= \tan^{-1} \left(\frac{3(2 - \epsilon^2)(2 - \lambda^2 - 2f\lambda^2) - 6h(2 - \epsilon^2)\lambda^2 \cos \delta - 4\sqrt{3}\epsilon\lambda(2 + h\lambda) \sin \delta}{2\sqrt{3}\epsilon(2 - \lambda^2 + 2f\lambda^2) + 4\sqrt{3}\epsilon\lambda(2 + h\lambda) \cos \delta - 6h(2 - \epsilon^2)\lambda^2 \sin \delta} \right) , \\
\beta_2 &= \tan^{-1} \left(\frac{3(2 - \epsilon^2)(1 + f\lambda^2) + 3h\lambda^2(2 - \epsilon^2) \cos \delta + 2\sqrt{3}\epsilon\lambda^2(h - 2f\lambda) \sin \delta}{2\sqrt{2}\epsilon(f\lambda^2 - 1) + 2\sqrt{3}\epsilon\lambda^2(h + 2f\lambda) \cos \delta - 3h\lambda^2(2 - \epsilon^2) \sin \delta} \right) . \quad (35)
\end{aligned}$$

From Eq. (34), the neutrino mixing parameters can be displayed as

$$\begin{aligned}
\sin^2 \theta_{12} &= \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} , & \sin^2 \theta_{23} &= \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} , \\
\sin \theta_{13} &= |U_{e3}| , & \delta_{CP} &= \alpha_1 - \alpha_3 . \quad (36)
\end{aligned}$$

It follows from Eqs. (34) and (36) that the solar neutrino mixing angle θ_{12} can be approximated, up to order λ^3 and ϵ^2 , as

$$\begin{aligned}
\sin^2 \theta_{12} &\simeq \frac{1}{3} + \frac{2\epsilon^2}{9} + \frac{2\lambda}{3} \left(\cos \delta + \frac{\epsilon \sin \delta}{\sqrt{3}} + \frac{\epsilon^2 \cos \delta}{3} \right) \\
&+ \frac{\lambda^2}{3} \left(\frac{1}{2} + \frac{2\epsilon \sin 2\delta}{\sqrt{3}} - \frac{\epsilon^2}{3}(3 + 4 \cos^2 \delta) \right) \\
&+ \frac{\lambda^3}{3} \left(2h - \frac{\epsilon \sin \delta}{\sqrt{3}} + \frac{\epsilon^2}{3}(2h - 7 \cos \delta) \right) . \quad (37)
\end{aligned}$$

This indicates that the deviation from $\sin^2 \theta_{12} = 1/3$ becomes small when $\cos \delta$ approaches to zero and the magnitude of ϵ is less than λ . Since it is the first column of V_L^ℓ that makes the major contribution to $\sin^2 \theta_{12}$, this explains why we need a phase of order 90° for the element $(V_L^\ell)_{21}$: When $|\sin \delta| \approx 1$, the present data of the solar mixing angle can be accommodated even for a large $|\epsilon|$ (but less than λ). The behavior of $\sin^2 \theta_{12}$ as a function of δ is plotted in Fig. 1 where the horizontal dashed lines denote the upper and lower bounds of the experimental data in 3σ ranges. The allowed regions for δ (in radian) lie in the ranges of $1.45 \lesssim \delta \lesssim 2.17$ and $4.17 \lesssim \delta \lesssim 4.91$, recalling that the QM phase is $\delta_{\text{QM}} = 1.56$.

Likewise, the atmospheric neutrino mixing angle θ_{23} comes out as

$$\begin{aligned}\sin^2 \theta_{23} \simeq & \frac{1}{2} - \frac{\epsilon\lambda}{\sqrt{3}} \left(\sin \delta - \frac{\epsilon \cos \delta}{\sqrt{3}} \right) \\ & - \lambda^2 \left(\frac{1}{4} + f + h \cos \delta + \epsilon \frac{2h \sin \delta}{\sqrt{3}} + \frac{2\epsilon^2}{3} (1 - f - h \cos \delta - \cos 2\delta) \right) \\ & - \lambda^3 \epsilon \left(\frac{\sin \delta}{2\sqrt{3}} (3 + 4h \cos \delta) - \epsilon \left[\frac{3 - 8f}{6} \cos \delta + h - 2h \cos^2 \delta \right] \right) .\end{aligned}\quad (38)$$

Fig. 1 shows a small deviation from the TBM atmospheric mixing angle with $\theta_{23} < 45^\circ$ for $0 < |\epsilon| < \lambda$. Owing to the absence of corrections to the first order of λ or ϵ in Eq. (38), the deviation from the maximal mixing of θ_{23} comes mainly from the terms associated with λ^2 or $\epsilon\lambda$. Especially, for $\sin \delta \approx 1$ we have the approximation $\sin^2 \theta_{23} - \frac{1}{2} \approx -\frac{\epsilon\lambda}{\sqrt{3}} - \lambda^2(f + \frac{1}{4})$, which implies $\sin^2 \theta_{23} < 1/2$ for $0 < |\epsilon| < \lambda$. We see from Fig. 1 that $\sin^2 \theta_{23}$ lies in the ranges $0.43 < \sin^2 \theta_{23} < 0.45$ for $0 \leq |\epsilon| \lesssim 0.1$.

The reactor mixing angle θ_{13} now reads

$$\begin{aligned}\sin^2 \theta_{13} = & \frac{2\epsilon\lambda \sin \delta}{\sqrt{3}} + \frac{2\epsilon^2}{3} (1 - \lambda \cos \delta) \\ & + \lambda^2 \left(\frac{1}{2} - \epsilon^2 \right) - \lambda^3 \epsilon \left(\frac{\sin \delta}{\sqrt{3}} + \frac{2h\epsilon}{3} - \frac{\epsilon \cos \delta}{3} \right) .\end{aligned}\quad (39)$$

Evidently, $\sin \theta_{13}$ depends considerably on the parameters λ and ϵ . Thus, we have a non-vanishing θ_{13} with a central value of $\sin \theta_{13} = \lambda/\sqrt{2}$ or $\theta_{13} = 9.2^\circ$ for $\epsilon = 0$ [13]. Note that the size of the unknown parameter ϵ is constrained by the plot of $\sin \theta_{13}$ versus δ in Fig. 1 where the horizontal dot-dashed lines represent the present T2K data for the normal neutrino mass hierarchy. For a negative value of ϵ , the plot for $\sin \theta_{13}$ versus δ is flipped upside-down. Assuming $\rho > 0$, we see from Eq. (23) that a positive (negative) value of ϵ leads to a normal (inverted) neutrino mass spectrum. For example, we find $\frac{\lambda}{\sqrt{2}} \leq \sin \theta_{13} \lesssim 0.22$ ($0.07 \lesssim \sin \theta_{13} \leq \frac{\lambda}{\sqrt{2}}$) for $\delta = 1.56$ and $\epsilon \leq 0.08$ ($\epsilon \geq -0.11$).

Leptonic CP violation can be detected through the neutrino oscillations which are sensitive to the Dirac CP -phase δ_{CP} , but insensitive to the Majorana phases in U_{PMNS} [22]. It follows from Eqs. (35) and (36) that the Dirac phase $\delta_{CP} = \alpha_1 - \alpha_3$ has the expression

$$\delta_{CP} = \tan^{-1} \left(\frac{\sqrt{3}\lambda \{-2 \cos \delta + \lambda \cos 2\delta + \lambda^2 \cos \delta + \sqrt{3}\epsilon\lambda \sin 2\delta\}}{2\sqrt{3}\lambda \sin \delta \{1 - \lambda \cos \delta - \frac{\lambda^2}{2}\} - 4\epsilon \{1 - \lambda \cos \delta - \frac{3}{2}\lambda^2 \cos^2 \delta - \lambda^3(h - \frac{\cos \delta}{2})\}} \right) ,\quad (40)$$

where terms of order $\epsilon^3, \lambda^4, \epsilon^2\lambda^2$ have been neglected in both numerator and denominator.

Assuming $\rho > 0$, we show in Table I the predictions for δ_P and θ_{13} as a function of ϵ , where we have used the central values of Eq. (31).

TABLE I: Predictions of δ_{CP} and θ_{13} as a function of ϵ in the case of $V_L^{\ell\dagger} = V_{QM}$.

ϵ	δ_{CP} [deg.]	θ_{13} [deg.]
$-0.012 \sim 0.08$	$-173.6 \sim -169$	$9.4 \sim 5.8$
$-0.11 \sim -0.012$	$184.6 \sim 186.4$	$14.4 \sim 9.4$

To see how the parameters are correlated with low energy CP violation measurable through neutrino oscillations, let us consider the leptonic CP violation parameter defined through the Jarlskog invariant $J_{CP} \equiv \text{Im}[U_{e1}U_{\mu 2}U_{e2}^*U_{\mu 1}^*] = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta_{CP}$ [23] which is expressed as

$$J_{CP} = -\frac{\epsilon}{3\sqrt{3}} - \frac{\lambda}{6} \left(\sin \delta + \epsilon \frac{4 \cos \delta}{\sqrt{3}} - 2\epsilon^2 \sin \delta \right) - \frac{\lambda^2}{9} \left(\sin \delta (h + \cos \delta) - \epsilon \sqrt{3} (1 + f - \cos 2\delta + h \cos \delta) - \epsilon^2 (h + 4 \cos \delta) \sin \delta \right). \quad (41)$$

We see from the above equation that J_{CP} is strongly correlated with ϵ and δ for the fixed values of λ , h and f . As long as $\epsilon \neq 0$ (associated with the neutrino part) or $\lambda \neq 0$ (associated with the charged lepton part), J_{CP} has a non-vanishing value, indicating a signal of CP violation. Eq. (41) could be approximated as $J_{CP} \approx -\frac{\epsilon}{3\sqrt{3}} - \frac{\lambda}{6} \sin \delta$. The behavior of J_{CP} is plotted in Fig. 1 as a function of δ . When $\sin \delta \approx 1$, it is reduced to $J_{CP} \approx -\frac{\epsilon}{3\sqrt{3}} - \frac{\lambda}{6} \leq -\frac{\lambda}{6} (\geq -\frac{\lambda}{6})$ for $\epsilon > 0$ ($\epsilon < 0$). Assuming $\rho > 0$, we find $-0.050 \lesssim J_{CP} \lesssim -0.037$ ($-0.037 \lesssim J_{CP} \lesssim -0.017$) for $\epsilon \leq 0.08$ ($\epsilon \geq -0.11$) and $\delta = 1.56$.

(ii) $V_L^\ell = V_{QM}$

The resulting leptonic mixing matrix in this case can be expressed, up to order of λ^3 and

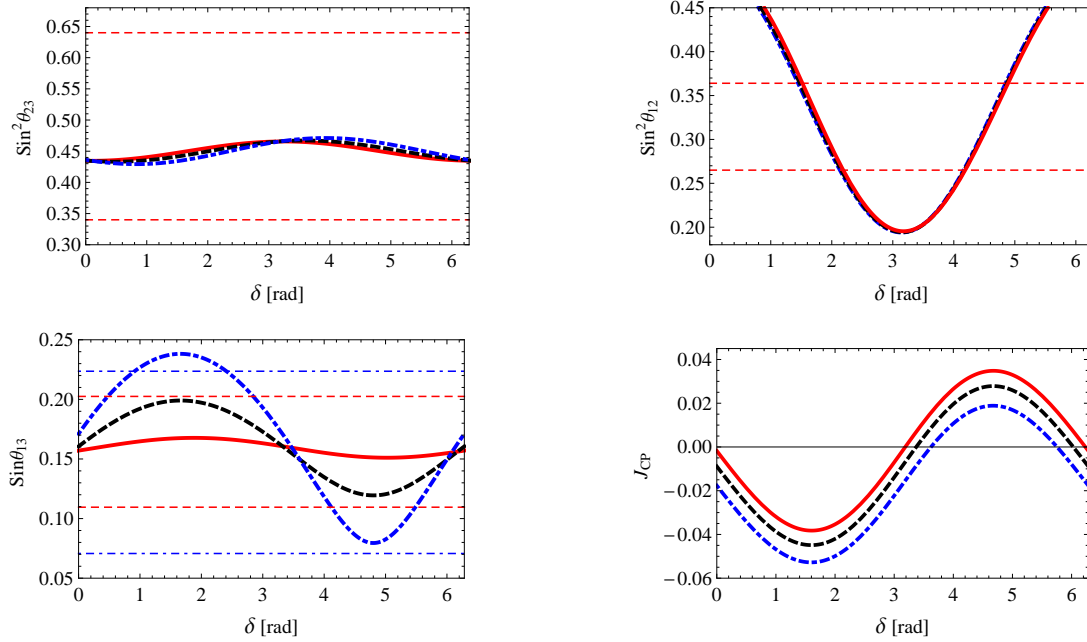


FIG. 1: Mixing angles $\sin^2 \theta_{23}$, $\sin^2 \theta_{12}$, $\sin \theta_{13}$ and the Jarlskog invariant J_{CP} as a function of the CP -odd phase δ for case (i), where the solid, dashed and dot-dashed curves are for $\epsilon = 0.01, 0.05$ and 0.1 , respectively, the horizontal dashed lines denote the upper and lower bounds in 3σ set by Fogli *et al.* (see Eq. (6)) and use of $\lambda = 0.2254$, $f = 0.749$, $h = 0.309$ has been made. In the plot of $\sin \theta_{13}$ versus δ , the horizontal dot-dashed lines represent the present data of T2K Collaboration for the normal neutrino mass hierarchy, see Eq. (8).

ϵ^2 , as

$$\begin{aligned}
U_{\text{PMNS}}^{(\text{ii})} &= U_L^\dagger U_\nu(\pi/4 + \epsilon) = V_{\text{QM}}^\dagger U_{\text{TBM}} W \\
&= U_{\text{TBM}} + \begin{pmatrix} -\frac{\epsilon^2}{2}\sqrt{\frac{2}{3}} & 0 & \epsilon\sqrt{\frac{2}{3}} \\ \frac{i\epsilon}{\sqrt{2}} + \frac{\epsilon^2}{2\sqrt{6}} & 0 & -\frac{\epsilon}{\sqrt{6}} + \frac{i\epsilon^2}{2\sqrt{2}} \\ -\frac{i\epsilon}{\sqrt{2}} + \frac{\epsilon^2}{2\sqrt{6}} & 0 & -\frac{\epsilon}{\sqrt{6}} - \frac{i\epsilon^2}{2\sqrt{2}} \end{pmatrix} \\
&+ \lambda \begin{pmatrix} \frac{e^{i\delta} - \lambda - f e^{i\delta} \lambda^2}{\sqrt{6}} & \frac{e^{i\delta} + \frac{1}{2}\lambda - f e^{i\delta} \lambda^2}{\sqrt{3}} & \frac{i e^{i\delta} (1 + f \lambda^2)}{\sqrt{2}} \\ \frac{2e^{-i\delta} + (\frac{1}{2} + f + h e^{-i\delta})\lambda}{\sqrt{6}} & \frac{e^{-i\delta} - (\frac{1}{2} + f + h e^{-i\delta})\lambda}{\sqrt{3}} & \frac{i(\frac{1}{2} - f - h e^{-i\delta})\lambda}{\sqrt{2}} \\ -\frac{(f + h e^{i\delta})\lambda - 2\lambda^2 h}{\sqrt{6}} & \frac{(f + h e^{i\delta})\lambda + h \lambda^2}{\sqrt{3}} & -\frac{i(f + h e^{i\delta})\lambda}{\sqrt{2}} \end{pmatrix} \\
&+ \lambda \epsilon \begin{pmatrix} -\frac{i e^{i\delta} (1 + f \lambda^2)}{\sqrt{2}} + \frac{-e^{i\delta} \epsilon + \lambda \epsilon + f e^{i\delta} \lambda^2 \epsilon}{2\sqrt{6}} & 0 & \frac{e^{i\delta} - \lambda - f e^{i\delta} \lambda^2}{\sqrt{6}} + \frac{-e^{i\delta} \epsilon - i f e^{i\delta} \lambda^2 \epsilon}{2\sqrt{2}} \\ \frac{i(f + h e^{-i\delta} - \frac{1}{2})\lambda}{\sqrt{2}} - \frac{2e^{-i\delta} \epsilon + (\frac{1}{2} + f + h e^{-i\delta})\lambda \epsilon}{2\sqrt{6}} & 0 & \frac{2e^{-i\delta} + (\frac{1}{2} + f + h e^{-i\delta})\lambda}{\sqrt{6}} + \frac{i(f + h e^{-i\delta} - \frac{1}{2})\lambda \epsilon}{2\sqrt{2}} \\ \frac{i(f + h e^{i\delta})\lambda}{\sqrt{2}} + \frac{(f + h e^{i\delta})\lambda \epsilon - 2h \lambda^2 \epsilon}{2\sqrt{6}} & 0 & -\frac{(f + h e^{i\delta})\lambda + 2\lambda^2 h}{\sqrt{6}} + \frac{i(f + h e^{i\delta})\lambda \epsilon}{2\sqrt{2}} \end{pmatrix}.
\end{aligned} \tag{42}$$

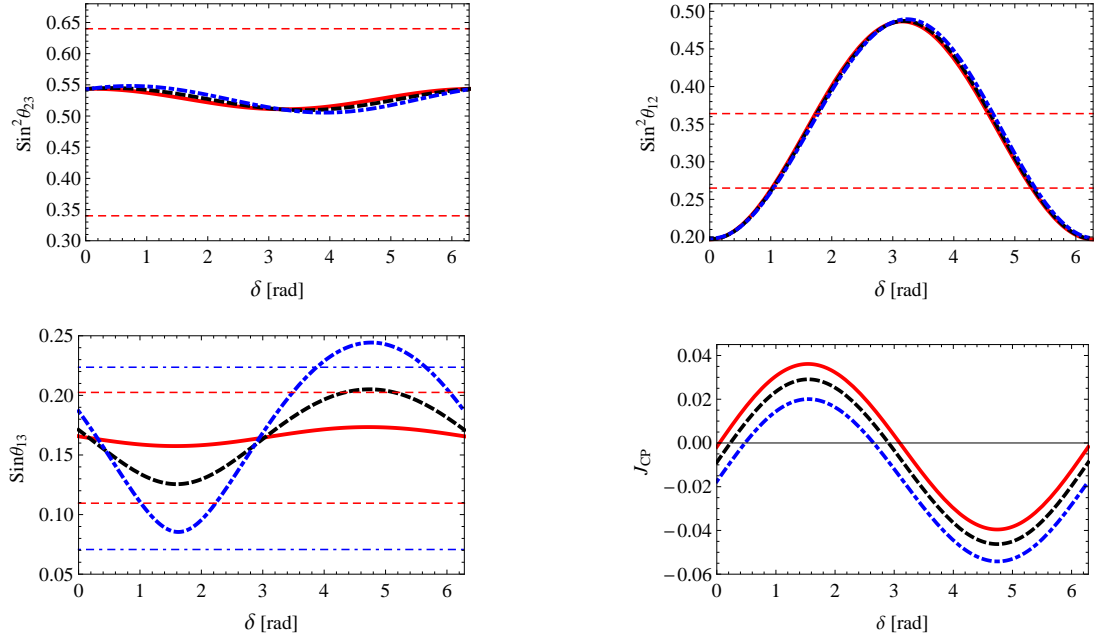


FIG. 2: Same as Fig. 1 except for case (ii).

Just as in case (i), the exact TBM is recovered when both ϵ and λ go to zero. With the help of Eqs. (36) and (42), the solar neutrino mixing angle θ_{12} can be approximated as

$$\begin{aligned} \sin^2 \theta_{12} \simeq & \frac{1}{3} + \frac{2\epsilon^2}{9} - \frac{2\lambda}{3} \left(\cos \delta + \frac{\epsilon \sin \delta}{\sqrt{3}} + \frac{\epsilon^2 \cos \delta}{3} \right) \\ & + \frac{\lambda^2}{3} \left(\frac{1}{2} + \frac{2\epsilon \sin 2\delta}{\sqrt{3}} - \frac{\epsilon^2}{3} (3 + 4 \cos^2 \delta) \right) \\ & + \frac{\lambda^3}{3} \left(2f \cos \delta + \frac{\epsilon \sin \delta}{\sqrt{3}} (1 - 2f) + \frac{\epsilon^2 \cos \delta}{3} (2f + 7) \right), \end{aligned} \quad (43)$$

which leads to, as in case (i), a tiny deviation from $\sin^2 \theta_{12} = 1/3$ when $\cos \delta \rightarrow 0$ and $\lambda > |\epsilon|$. As expected, since the second column related to ϵ in the matrix Eq. (42) is zero, the solar mixing angle is not affected to the first order of ϵ . Because of a minus sign in front of the $\lambda \cos \delta$ term, which constitutes the major correction to $\sin \theta_{12}$, the plot of $\sin^2 \theta_{12}$ versus δ (see Fig. 2) is turned upside-down, contrary to case (i). When $\sin \delta \approx 1$, the present data of the solar mixing angle are well accommodated even for a large $|\epsilon|$ (but less than λ). The allowed regions for δ lie in the ranges of $1.0 < \delta < 1.7$ and $4.5 < \delta < 5.3$. This indicates that when the CP -odd phase δ is near maximal, the data of $\sin^2 \theta_{12}$ can be easily accommodated in case (ii) but only marginally in case (i). Hence, the precise measurements of the solar mixing angle in future experiments will tell which scenario is more preferable.

From Eqs. (36) and (42), the atmospheric neutrino mixing angle θ_{23} comes out as

$$\begin{aligned} \sin^2 \theta_{23} \simeq & \frac{1}{2} + \frac{\epsilon \lambda}{\sqrt{3}} \left(\sin \delta + \frac{\epsilon \cos \delta}{\sqrt{3}} (1 - 2\sqrt{3}) \right) \\ & - \lambda^2 \left(\frac{1}{4} - f - h \cos \delta - \epsilon \frac{2h \sin \delta}{\sqrt{3}} + \frac{\epsilon^2}{3} (2 \sin^2 \delta + f + h \cos \delta) \right) \\ & + \frac{\lambda^3 \epsilon}{2} \left(\frac{\sqrt{3}}{3} \sin \delta (3 - 2f - 4h \cos \delta) \right. \\ & \left. - \frac{\epsilon}{3} \left[(1 + 2\sqrt{3} - 6f - 4h \cos \delta) \cos \delta + 8h \sin^2 \delta - 4h \right] \right). \end{aligned} \quad (44)$$

Fig. 2 shows a small deviation from the TBM atmospheric mixing angle with $\theta_{23} > 45^\circ$, recalling that $\theta_{23} < 45^\circ$ in case (i). It is thus crucial to have precise measurements of the atmospheric mixing angle in the future to see whether $\theta_{23} \leq 45^\circ$ or $\theta_{23} \geq 45^\circ$ in order to test different scenarios. For $\sin \delta \approx 1$ the deviation from the maximal mixing of θ_{23} is approximated as $\sin^2 \theta_{23} - \frac{1}{2} \approx \frac{\epsilon \lambda}{\sqrt{3}} + \lambda^2 (f - \frac{1}{4})$, which leads to $\sin^2 \theta_{23} > 1/2$ for $0 < |\epsilon| < \lambda$. The behavior of $\sin^2 \theta_{23}$ is plotted in Fig. 2 as a function of δ . Likewise, the reactor mixing angle θ_{13} can be written as

$$\begin{aligned} \sin^2 \theta_{13} = & \frac{2\epsilon^2}{3} + \frac{2\epsilon \lambda \sin \delta}{3} (\epsilon \cos \delta - \sqrt{3} \sin \delta) + \lambda^2 \left(\frac{1}{2} - \epsilon^2 \right) \\ & + \lambda^3 \epsilon \left(\frac{\sin \delta (1 - 2f)}{\sqrt{3}} - \frac{\epsilon \cos \delta (1 + 2f)}{3} \right). \end{aligned} \quad (45)$$

We find $0.07 \lesssim \sin \theta_{13} \leq \frac{\lambda}{\sqrt{2}}$ ($\frac{\lambda}{\sqrt{2}} \leq \sin \theta_{13} \lesssim 0.22$) for $\delta = 1.56$ and $\epsilon \leq 0.12$ ($\epsilon \geq -0.07$).

The Dirac phase δ_{CP} has the expression

$$\begin{aligned} \delta_{CP} = & \tan^{-1} \left(\frac{\lambda \{ \sqrt{3} (2 - \epsilon^2) (1 - f \lambda^2) \sin \delta - 6\epsilon (1 + f \lambda^2) \cos \delta \}}{\sqrt{3} (2 - \epsilon^2) \{ 2 - \lambda^2 + (1 - f \lambda^2) \lambda \cos \delta \} + 6\epsilon \lambda (1 + f \lambda^2) \sin \delta} \right) \\ & - \tan^{-1} \left(\frac{\lambda \{ 2\sqrt{3} \epsilon (1 - f \lambda^2) \sin \delta + 3(2 - \epsilon^2) (1 + f \lambda^2) \cos \delta \}}{3\lambda (\epsilon^2 - 2) (1 + f \lambda^2) \sin \delta + 2\sqrt{3} \epsilon (2 - \lambda^2 + (1 - f \lambda^2) \cos \delta)} \right). \end{aligned} \quad (46)$$

Assuming $\rho > 0$, we show in Table II the predictions for δ_{CP} and θ_{13} as a function of ϵ , where we have focused on the central values of Eq. (31).

The strength of CP violation J_{CP} can be expressed in a similar way to Eq. (41)

$$\begin{aligned} J_{CP} = & -\frac{\epsilon}{3\sqrt{3}} + \frac{\lambda}{6} \left(\sin \delta + \epsilon \frac{4 \cos \delta}{\sqrt{3}} - 2\epsilon^2 \sin \delta \right) \\ & + \frac{\lambda^2}{9} \left(\sin \delta (h - \cos \delta) + \epsilon \sqrt{3} (1 - f - \cos 2\delta - h \cos \delta) - \epsilon^2 (h - 4 \cos \delta) \sin \delta \right), \end{aligned} \quad (47)$$

TABLE II: Predictions of δ_{CP} and θ_{13} as a function of ϵ in the case of $V_L^\ell = V_{QM}$.

ϵ	δ_{CP} [deg.]	θ_{13} [deg.]
$0 \sim 0.08$	$-7.1 \sim -5.2$	$9.1 \sim 12.6$
$-0.11 \sim 0$	$-15.6 \sim -7.1$	$4.1 \sim 9.1$

which can be approximated as $J_{CP} \approx -\frac{\epsilon}{3\sqrt{3}} + \frac{\lambda}{6} \sin \delta$. When $\sin \delta \approx 1$, it is further reduced to $J_{CP} \approx -\frac{\epsilon}{3\sqrt{3}} + \frac{\lambda}{6} \leq \frac{\lambda}{6}$ ($\geq \frac{\lambda}{6}$) for $\epsilon > 0$ ($\epsilon < 0$). Assuming $\rho > 0$, we see from Fig. 2 that $0.014 \lesssim J_{CP} \lesssim 0.037$ ($0.037 \lesssim J_{CP} \lesssim 0.05$) for $\epsilon \leq 0.12$ ($\epsilon \geq -0.07$) and $\delta = 1.56$.

IV. CONCLUSION

In their original work, Harrison, Perkins and Scott proposed simple charged lepton and neutrino mass matrices that lead to the tribimaximal mixing U_{TBM} . In this paper we considered a general extension of the mass matrices so that the lepton mixing matrix becomes $U_{PMNS} = V_L^{\ell\dagger} U_{TBM} W$. Hence, corrections to the tribimaximal mixing arise from both charged lepton and neutrino sectors: the charged lepton mixing matrix V_L^ℓ measures the deviation of from the trimaximal form and the W matrix characterizes the departure of the neutrino mixing from the bimaximal one. Following our previous work to assume a Qin-Ma-like parametrization V_{QM} for V_L^ℓ in which the CP -odd phase is approximately maximal, we study the phenomenological implications in two different scenarios: $V_L^\ell = V_{QM}^\dagger$ and $V_L^\ell = V_{QM}$. We found that both scenarios are consistent with the data within 3σ ranges. Especially, the predicted central value of the reactor neutrino mixing angle $\theta_{13} = 9.2^\circ$ is in good agreement with the recent T2K data. However, the data of $\sin^2 \theta_{12}$ can be easily accommodated in the second scenario but only marginally in the first one. Hence, the precise measurements of the solar mixing angle in future experiments will test which scenario is more preferable. The leptonic CP violation characterized by the Jarlskog invariant J_{CP} is generally of order 10^{-2} .

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